Introduction

Algebra is the most basic technique throughout Mathematics. We will explore each technique systematically, including expansion and factorisation, algebraic fractions, solving equation(s) and completing the square.

✧ Lesson Aim:

- Expansion of Binomial Products
- Factorisation
  - Binomials
  - Trinomials
  - Four Terms
- Operations in Algebraic Fractions
  - Addition/Subtraction

1. Expansion of Binomial Products

Revision: perfect squares:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Example 1.1

(a) Prove that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
(b) Hence, by using the fact that $(a-b)^3 = [a+(-b)]^3$, prove that $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

Perfect cubes:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

NOTE:

The power of a decreases by 1, as the power of b increases by 1.

NOTE:

For the negative flip, it's $(-b)^n$, when n is **ODD**, the term is negative, when n is **EVEN**, the term is positive.

Step 1. Write the terms of a and b with the corresponding pattern first.
Step 2. Fill in the coefficients.

Example 1.2

Expand the following expressions:

(a) $(x+2)^3$

(b) $(3x - y)^3$
(c) $(5x - 2y)^3$

Exercise 1.1

Simplify the following expressions:

(a) $(3x+1)^3+(2x-3y)^2$
(b) $(a+3b)^3+(3a-b)^3$
(c) $(s+2)^3-(2-s)^3$

2. Factorisation

Step 1. **ALWAYS** find the highest common factors of all the terms first.
Step 2. If there are two terms, try difference of squares, difference or sum of cubes.
Step 3. If there are three terms, try quadratic factorisation.
Step 4. If there are four terms, try grouping in pairs.

Binomials

Revision: difference of two squares:

$$a^2 - b^2 = (a - b)(a + b)$$

Example 2.1

(a) Prove that $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

(b) Hence, by using the fact that $a^3 + b^3 = a^3 - (-b)^3$, prove that $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

Difference of two cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Sum of two cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

NOTE:

$(a - b)$ first, then from a^2, the power of a decreases by 1, as the power of b increases by 1.

NOTE:

For the negative flip, it's $(-b)^n$, when n is **ODD**, the term is negative, when n is **EVEN**, the term is positive.

Example 2.2

Factorise the following expressions as much as possible:

(a) $x^3 - 8$
(b) $216 + a^3$

(c) $64a^3b^3 + 1$
(d) $81m^3 + 3n^6$

Exercise 2.1

Factorise the following expressions as much as possible:

(a) $25y - y^3$
(b) $a^4 - b^4$
(c) $x^6 - y^6$

Trinomials

Step 1. Find two factors that gives a **PRODUCT OF ac** and a **SUM OF b**.
Step 2. Construct two brackets and split ax^2 into $\_x$ and $\_x$, the two numbers must be able to divide the two factors correspondingly.
Step 3. **CROSS** multiply to fill up the second terms and multiply to get the two factors.

Example 2.3

Factorise the following expressions as much as possible:

(a) $x^2 - 3x - 4$
(b) $6x^2 - 5x - 4$

(c) $a^2 + 12ac - 28c^2$

Exercise 2.2

Factorise the following expressions as much as possible:

(a) $2x^2 + 5x + 2$
(b) $6x^2 - 11x + 3$
(c) $9x^2 - 6x - 8$
(d) $12 - 29x + 14x^2$

(e) $6d - 4 - 2d^2$
(f) $9x^2y^2 - 12xy - 5$
(g) $6s^2 - 11st - 10t^2$

Grouping in Pairs

Step 1. Factorise in pairs.
Step 2. Factorise the **COMMON FACTOR**.

Example 2.5

Factorise the following expressions as much as possible:

(a) $x^2 + 2x + 6xy + 12y$
(b) $ab^2 - ab + b - 1$
(c) $x^3 + x^2 - x - 1$

Exercise 2.3

Factorise the following expressions as much as possible:

(a) $12xy - 9x - 16y + 12$
(b) $x^4 - x^2 - x - 1$
(c) $3t - ax^2 + tx^2 - 3a$

3. Operations in Algebraic Fractions

Additions/Subtractions

Step 1. Factorise all the denominators.
Step 2. Put all the fractions over the lowest common denominator.
Step 3. Add/subtract the numerators.

Example 3.1

Simplify the following fractions

(a) $\frac{5x}{6} + \frac{7x-1}{8}$

(b) $\dfrac{3}{x^2+2x}-\dfrac{2}{x^2-4}$

(c) $\dfrac{1}{x^2-4x+3}+\dfrac{1}{x^2-5x+6}-\dfrac{1}{x^2-3x+2}$

Exercise 3.1

Simplify the following fractions

(a) $\dfrac{a^2}{a^3+b^3}+\dfrac{a-b}{a^2-ab+b^2}+\dfrac{1}{a+b}$